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UNIVERSITY OF SASKATCHEWAN
ELECTRICAL ENGINEERING 455.3

Assignment Quiz 3
October 15, 2001

Instructor: B.L. Daku
Time: 15 minutes
Aids: None

Name:
Student Number:

$\cos x = \frac{e^{jx} + e^{-jx}}{2}$

$\sin x = \frac{e^{jx} - e^{-jx}}{2j}$

$e^{jx} = \cos x \pm j \sin x$

1. Consider an LTI system with frequency response

$$H(e^{j\omega}) = e^{-j(\omega/2+\pi/4)}, \quad -\pi < \omega \le \pi$$

Determine $y[n]$, the output of this system, if the input is

$$x[n] = \cos\left(\frac{15\pi n}{4} - \frac{\pi}{3}\right)$$

for all n.

$\frac{8\pi}{4} = 2\pi$

$\frac{15\pi}{4} = \frac{7\pi}{4} = -\frac{\pi}{4}$

$\omega = -\frac{\pi}{4}$

$H(e^{j\omega}) = e^{-j(\frac{\omega}{2} + \frac{\pi}{4})}$

$x[n] = \cos\left(\frac{15\pi n}{4} - \frac{\pi}{3}\right)$

$= \frac{1}{2}\left(e^{j(\frac{-\pi n}{4} - \frac{\pi}{3})} + e^{-j(\frac{-\pi n}{4} - \frac{\pi}{3})}\right)$

$= \frac{1}{2}\left(e^{-j(\frac{\pi n}{4} + \frac{\pi}{3})} + e^{+j(\frac{\pi n}{4} + \frac{\pi}{3})}\right)$

$y_1[n] + y_2[n]$

$= \frac{1}{2}\left[e^{-j(\frac{-\pi}{8} + \frac{\pi}{4})} e^{-j(\frac{\pi n}{4} + \frac{\pi}{3})}\right]$

$= \frac{1}{2}\left[e^{-j(-\frac{\pi}{8} + \frac{\pi}{4})} e^{-j(\frac{\pi n}{4} + \frac{\pi}{3})}\right]$

$= \frac{1}{2}\left[e^{-j(\frac{\pi}{8})} e^{-j(\frac{\pi n}{4} + \frac{\pi}{3})}\right]$

$= \frac{1}{2}\left[e^{-j(\frac{\pi n}{4} + \frac{\pi}{3} + \frac{\pi}{8})}\right]$

$= \frac{1}{2}\left[e^{-j(\frac{\pi n}{4} + \frac{11\pi}{24})}\right]$

$y_2[n] = \frac{1}{2}\left[e^{-j(\frac{\pi}{8} + \frac{\pi}{4})} e^{j(\frac{\pi n}{4} + \frac{\pi}{3})}\right]$

$= \frac{1}{2}\left[e^{-j(\frac{3\pi}{8})} e^{j(\frac{\pi n}{4} + \frac{\pi}{3})}\right]$

$= \frac{1}{2}\left[e^{-j(\frac{3\pi}{8})} e^{j(\frac{\pi n}{4} + \frac{\pi}{3})}\right]$

$= \frac{1}{2}\left[e^{j(\frac{\pi n}{4} - \frac{\pi}{24})}\right]$

$\frac{3\pi}{8}$

$\frac{\pi}{3} - \frac{3\pi}{8}$

$\frac{8\pi}{24} - \frac{9\pi}{24}$

$-\frac{\pi}{24}$

$y[n] = \frac{1}{2}\left[e^{-j(\frac{\pi n}{4} + \frac{11\pi}{24})} + e^{j(\frac{\pi n}{4} - \frac{\pi}{24})}\right]$

$= \frac{1}{2} e^{-\frac{5}{24}}\left[e^{-j(\frac{\pi n}{4} + \frac{6\pi}{24})} + e^{j(\frac{\pi n}{4} + \frac{6\pi}{24})}\right]$

$= \frac{1}{2} e^{-\frac{5}{24}}\left[2\cos\left(\frac{\pi n}{4} + \frac{6\pi}{24}\right)\right]$

$\boxed{y[n] = e^{-\frac{5}{24}}\left(\cos \frac{\pi}{4}(n+1)\right)}$

$-\frac{6\pi}{24}$

$e^{-\frac{5}{24}}$

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$e^{jx} = \cos x \pm j \sin x$

1. The impulse response for an LTI system is given by

$$h[n] = e^{j\pi/4}\delta[n-2].$$

(a) Determine the frequency response, $H(e^{j\omega})$, for the system.

(b) Determine the output $y[n]$ given the input

$$x[n] = 2 + 4\cos\left(\frac{\pi n}{2} - \frac{3\pi}{10}\right).$$

Simplify $y[n]$, making it a function of a cosine.

$2\cos jx =$

a) $H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} h[k]\, e^{-j\omega k}$

$n = 2$

$= e^{j\frac{\pi}{4}} e^{-2j\omega}$

$= \boxed{e^{j(\frac{\pi}{4} - 2\omega)}}$

$\frac{40\pi}{20}$

b) $y[n] = H(e^{j\omega})\, x[n]$

$\omega = \frac{\pi}{2}$

$\cos x = \frac{e^{jx} + e^{-jx}}{2}$

$x[n] = 2 + 2e^{j(\frac{\pi n}{2} - \frac{3\pi}{10})} + 2e^{-j(\frac{\pi n}{2} - \frac{3\pi}{10})}$

$= 2[1 + e^{j(\frac{\pi n}{2} - \frac{3\pi}{10})} + e^{-j(\frac{\pi n}{2} - \frac{3\pi}{10})}]$

2 is a constant
DC term
$\omega = 0$

$y[n] = 2(e^{j(\frac{\pi}{4} - 0)}) + 2\left[e^{j(\frac{\pi n}{2} - \frac{3\pi}{10})} e^{j(\frac{\pi}{4} - 2(\frac{\pi}{2}))} + e^{-j(\frac{\pi n}{2} - \frac{3\pi}{10})} e^{j(\frac{\pi}{4} - 2(-\frac{\pi}{2}))}\right]$

$= 2e^{j\frac{\pi}{4}} + 2\left[e^{j(\frac{\pi n}{2} - \frac{3\pi}{10} - \frac{3\pi}{4})} + e^{j(\frac{-\pi n}{2} - \frac{3\pi}{10} + \frac{5\pi}{4})}\right]$

$= 2e^{j\frac{\pi}{4}} + 2\left[e^{j(\frac{\pi n}{2} - \frac{21\pi}{20})} + e^{-j(\frac{\pi n}{2} + \frac{19\pi}{20})}\right]$

$\frac{-21\pi}{20} = \frac{19\pi}{20}$

$+ 2\left[e^{j(\frac{\pi n}{2} + \frac{19\pi}{20})} + e^{-j(\frac{\pi n}{2} + \frac{19\pi}{20})}\right]$

$= 2e^{j\frac{\pi}{4}} + 4\left(\cos\left(\frac{\pi n}{2} + \frac{19\pi}{20}\right)\right) e^{j\pi/4}$

$\frac{3\pi}{10} - \frac{3\pi}{4}$

$\frac{12\pi}{40} - \frac{30\pi}{40}$